SOLID STATE PHYSICS I

EXAM August 21, 2002

◇ Do not forget to write your full name and student number on each sheet.
◇ Please use separate sheets for each of the problems.
◇ The answers may be given in dutch

Problem 1

Phonons and thermal properties

a) Certain crystals have phonon dispersion relations showing two types of branches: The so-called acoustical and optical branches. What is the condition for both types of branches to be present ?
b) Describe the main difference between these 2 types.
c) Give the definition of the group velocity, and explain how to determine the sound velocity.
d) Describe the difference between transversal and longitudinal phonons.
e) Make a sketch of the phonon density of states for the Einstein model, the Debye model, and for a real crystal.
f) Derive the relation for the phonon specific heat according to the Einstein model, consider the high and the low temperature behavior.
g) Describe Umklapp processes. How do these influence the thermal conductivity ?

Problem 2

Aluminum (Al) has a fcc structure, the lattice constant is 4.05 Å.

a) Calculate the volume of the Wigner-Seitz cell of Al.
b) Derive an expression for the density of states, $D(E)$ of the electrons in three dimensions using the free electron model.
c) Derive an expression for the value of the Fermi energy E_{F} using the free electron model. Note that Al has 3 conduction electrons per atom.
d) Show, using

$$U_{\mathrm{e}} = \int_0^{E_{\mathrm{F}}} f(\epsilon) D(\epsilon) \epsilon \, d\epsilon,$$

where $f(\epsilon)$ is the Fermi-Dirac function, and U_{e} the total energy of the electrons, that for $T = 0$ K the average energy per electron is $3/5\ E_{\mathrm{F}}$.

e) For $T > 0$ K the energy of the electrons can be approximated as

$$U_{\mathrm{e}} = \frac{3}{5} E_{\mathrm{F}} \left[1 + \frac{5\pi^2}{12} \left(\frac{k_{\mathrm{b}} T}{E_{\mathrm{F}}} \right)^2 \right],$$

and the energy of the phonons as

$$U_{\mathrm{p}} = C \frac{(k_{\mathrm{b}} T)^4}{(\hbar \omega_{\mathrm{D}})^3},$$

with $C \approx 1$.
Use this to derive an equation for the total heat capacity (*i.e.* including both the electronic and the phononic contribution.)

f) At which temperature does the phonon contribution begin to exceed the electronic contribution ?

Problem 3

Consider the following linear chain of Ni and Co atoms:
... Ni Co Ni Co Ni Co Ni Co Ni Co ...
We assume that both type of atoms have a Curie type susceptibility of the form $\chi_i = M_i/B_i = C_i/T$, where M_i is the sublattice magnetization of the Ni or Co lattice. We also assume that there is only a ferromagnetic exchange interaction between nearest neighbor atoms (*i.e.* between Ni and Co atoms).

a) Assume that the mean field constant λ for the Weiss (or exchange) field of the Ni atoms on the Co sites, and for the Co atoms on Ni sites are equal. Derive the two expressions for the total effective field on the Ni and Co sites in the presence of an external magnetic field B.

b) Derive an expression for the total susceptibility (χ). Give an expression for the temperature T_c at which χ diverges.

c) What happens below T_c ? Sketch the total magnetization $M(T)$ as a function of temperature below T_c.

d) Explain qualitatively what magnons are. Sketch the energy versus momentum dispersion of the magnons, assuming that the Curie constants of Ni and Co are equal ($C_{\mathrm{N}i} = C_{\mathrm{C}o}$).